The influence of an abrupt convergence on heat transfer in circular ducts

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The effect of an abrupt convergence on the local heat transfer coefficient at the entrance of a uniformly heated straight pipe has been investigated. Experiments were carried out with water for Reynolds numbers from 200 to 2000, and for Prandtl numbers 4 and 6. The experimental values of the local Nusselt number are considerably lower than expected and strongly Reynolds number dependent in the region near the inlet, whereas they become comparatively high further downstream. This behaviour is discussed in connection with the hypothesis of the occurrence of a separated flow at the inlet section of the pipe. These and several previous experiments, covering turbulent and transitional flows, have been considered and summarised to obtain a general view of the effect of this geometrical configuration on heat transfer

Key words: *laminar flow, entry effects, separation, heat transfer*

The heat transfer in pipes has been studied extensively for many years both theoretically and experimentally. Although many of the fundamental questions have now been solved, some features of the problem still remain incompletely understood. One such problem is the influence of the shape of the inlet section on the heat transfer rate at the wall.

While the experiments of Boelter *et al*[1] and Mills[2] have shown that the geometrical configuration of the entrance strongly affects the heat transfer and that the effect extends a considerable distance downstream in turbulent flow, no reference to the same problem in laminar flow has been found in the literature. This lack of interest may result from the generally accepted idea that the inlet disturbance should be damped in a short distance when *Re* is low. That this assumption might be invalid, at least for an entrance with a 90° angle sharp edge, has been suggested by Collins[3] when discussing experiments carried out by Martin and Fargie[4] and Butterworth and Hazell[5].

The only published experimental data on this subject are the few laminar convection results for a pipe following a 2:1 contraction reported by Ede *et al*[6]; the tests performed by Kemeny and Somers[7] and by Lawrence and Chato[8] are mainly concerned with mixed-convection phenomena, so that the influence of the abrupt convergence has been disregarded. Experiments with water flowing in a uniformly heated vertical pipe leading from a large reservoir have been performed by Giulianini *et al.*[9,10] for transitional and laminar flow and Barozzi *et al*[11,12] for laminar flow with a similar experimental apparatus. The measurements[9–12] have been compared, the test procedures discussed, and correlating equations for the results developed[13].

The main aim of this paper is to summarise the previous experimental results and to discuss the influence of an abrupt convergence on forced convection in pipes, on the basis of the 'separation hypothesis.' In fact, while the development of a 'vena contracta' at the inlet of a pipe following a contraction is well known in turbulent flow, even at low values of the contraction ratio, the same is not true for laminar flow. Nevertheless some experiments for laminar flow are quoted in the literature[14–17].

Water and glycerol solutions in a pipe with a contraction ratio γ equal to 2.5 have been studied by Astarita and Greco[14] who were able to sense the formation of a vena contracta by axial pressure distribution measurements at *Re* as low as 186. The data for the flow of glycerol solution in an 8:1 contraction, supplied by Sylvester and Rosen[15], clearly indicate the existence of separation at *Re* greater than 1000. Velocity measurements in a square conduit with a sharp entrance were made by Goldstein and Kreid[16], for *Re* as high as about 400, with air. Velocity fluctuations have been observed near the wall of the duct, at about 2.15 diameters from the inlet. Based on their own and some of the previous experiments, Rama Murthy and Boger[17] have suggested that a contraction ratio of 2 may be the maximum that can be used without the occurrence of separation in laminar flow. They have also shown that, even at this low value of γ, a vena contracta is formed when the flow is transitional.

Experimental apparatus

The general plan of the test loop is given in Fig 1. The test section consists of a 10 mm id (18 mm od) copper pipe 1.0 m long. The uniform heat flux condition at the inner wall is approximated by electrically

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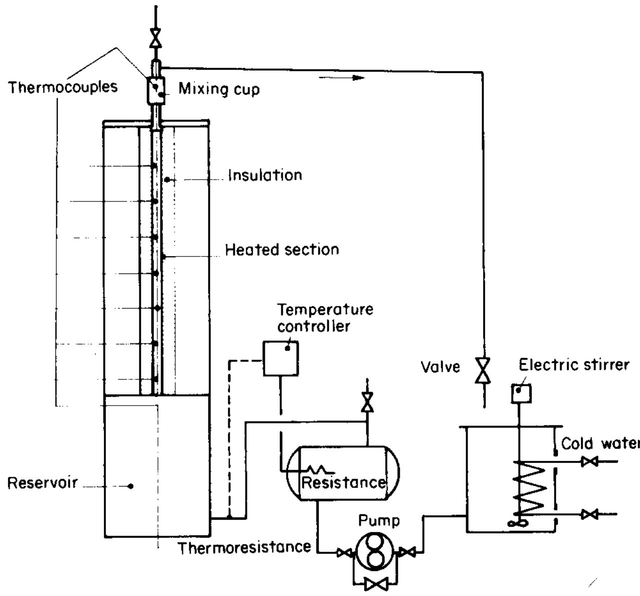

Fig 1 Schematic diagram of test installation

heating the measuring section, 0.76 m in length, via a calibrated constantan wire inserted into a helical groove on the external wall of the pipe. The entrance section and the connection of the pipe to the plexiglas cover of the cylindrical coaxial reservoir are shown in Fig 2. The diameter contraction ratio is 26.5. Twenty suitably spaced copper-constantan thermocouples sense the wall temperature along the test section. The bulk temperatures of the fluid before the inlet and after the mixing cup following the test section are also measured by thermocouples.

The pipe was insulated with 50 mm thick mineral wool and an external metal shield. The thickness of the insulation was calculated in order to reduce radial heat leakages to less than 2% of the supplied electrical power. Nevertheless, for low values of the Reynolds number, this limit might be exceeded in the end part of the test section. The heat loss from the top-end of the test section was controlled by a properly regulated guard resistance.

The experimental device and the measuring procedure has been described in detail elsewhere[11]. It is worthwhile pointing out, however, that the inner surface of the pipe was thoroughly cleaned before each of the tests reported here, since a rather fast deposition of metal oxides occurred, mainly immediately after the entrance[13].

Care was taken in the calibration of the thermocouples and their placing on the wall of the pipe. An overall precision of the temperature measurements better than 0.1 °C was obtained; nevertheless the final results, ie local Nusselt number values, are given with an accuracy of about 10%.

Analysis of the data

All the rests were performed in rigorously steady state conditions and the results were divided into two groups according to the Prandtl number, nominally 4 and 6. The inlet temperature was fixed between 35–38 °C for the first group and 20–23 °C for the second. The flow rate ranged between 1.4 and 15×10^{-6} m^3/s, and the heating flux at the wall between 3.6 and 10.8 kW/m^2.

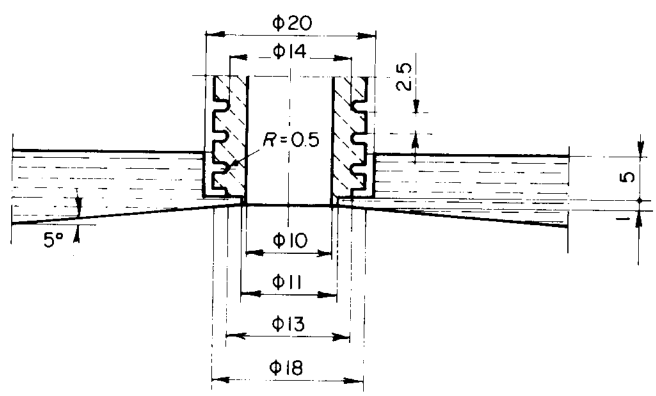

Fig 2 Connection of the test section to the reservoir cover

Notation

B	Coefficient in Eq 3.
c	Specific heat capacity of the fluid, J/kg K
D	Inner pipe diameter, m
D_r	Reservoir diameter, m
g	Gravitational acceleration, m/s^2
h	Convective heat transfer coefficient, W/m^2 K
l	Distance of the point of inflexion from the inlet, m
m	Flow rate, kg/s
Nu	Nusselt number $= hD/\lambda$
Pr	Prandtl number $= \mu c/\lambda$
q	Wall heat flux, W/m^2
Ra	Rayleigh number $= (g\beta D^4 q/4\lambda\nu^2)\, 1/Re$
Re	Reynolds number $= UD/\nu$
t	Temperature, K
U	Fluid mean velocity, m/s
x	Axial coordinate, m
x^*	Non-dimensional axial coordinate $= x/(D\,Re\,Pr)$
β	Coefficient of thermal expansion, K^{-1}
γ	Diameter contraction ratio $= D_r/D$
λ	Thermal conductivity of the fluid, W/m K
μ	Fluid dynamic viscosity coefficient, kg/ms
ν	Kinematic fluid viscosity coefficient, m^2/s

Subscripts

w	Wall
f	Fluid
o	Fluid at the inlet
x	Local
m	Mean
c	Constant properties

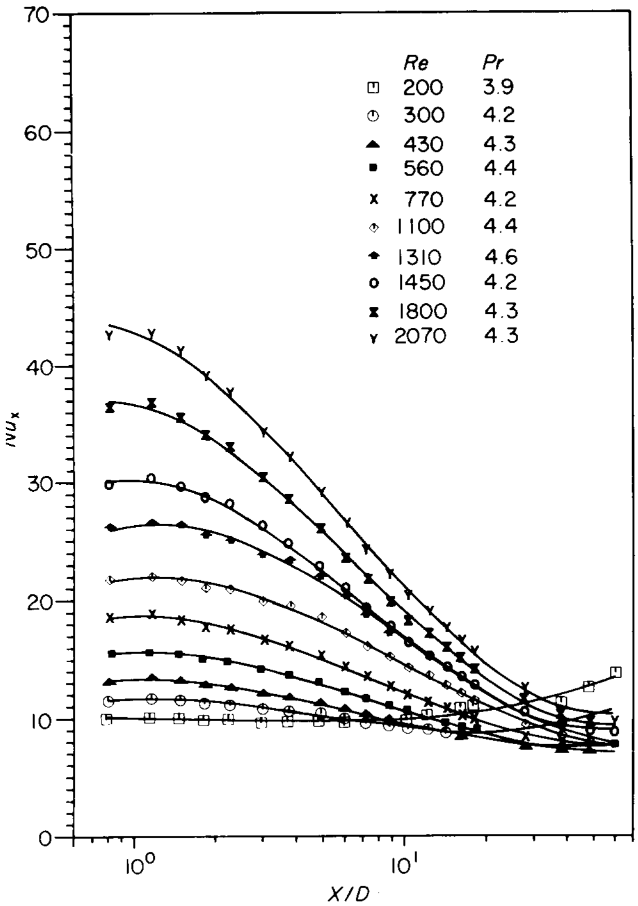

Fig 3 Local Nusselt number (Nu_x) *versus the dimensionless distance* (x/D) *from the entrance* (Pr ≅ *4*)

The local Nusselt number was obtained by the following relation:

$$Nu_x = \frac{hD}{\lambda} = \frac{D}{\lambda} \frac{q}{t_w - t_f} \tag{1}$$

where

$$t_f = t_0 + \frac{q\pi Dx}{mc} \tag{2}$$

Nu_x is plotted in Figs 3 and 4 against the dimensionless distance from the entrance, x/D. The solid lines show the least-squares fit of the experimental points.

Discussion

From the data in Figs 3 and 4, Nu_x versus x^* graphs have been plotted. Some of them are shown in Figs 5 and 6 where a few results markedly affected by natural convection have been suppressed. The solid line represents the Churchill and Ozoe correlation[18] (see their Eq (32)) for laminar flow in cylindrical pipes with uniform heat flux at the wall and flat profiles of velocity and temperature at the inlet.

Note that, at very low x^*, Nu_x is strongly Reynolds number dependent and considerably lower than expected from constant properties predictions. By comparing corresponding *Re* tests, Nu_x values are found to increase with *Pr*. Further downstream they tend to converge on a single line, which is higher than the theoretical one but with a similar trend, and show the expected decreasing dependence on *Pr*.

The influence of axial conduction, viscosity variation, free convection, and reference temperature, has been evaluated for all the data.

• The axial conduction flux along the wall has been computed numerically using the experimental values of wall temperature. The correction for the radial heat flux has never been found to be greater than

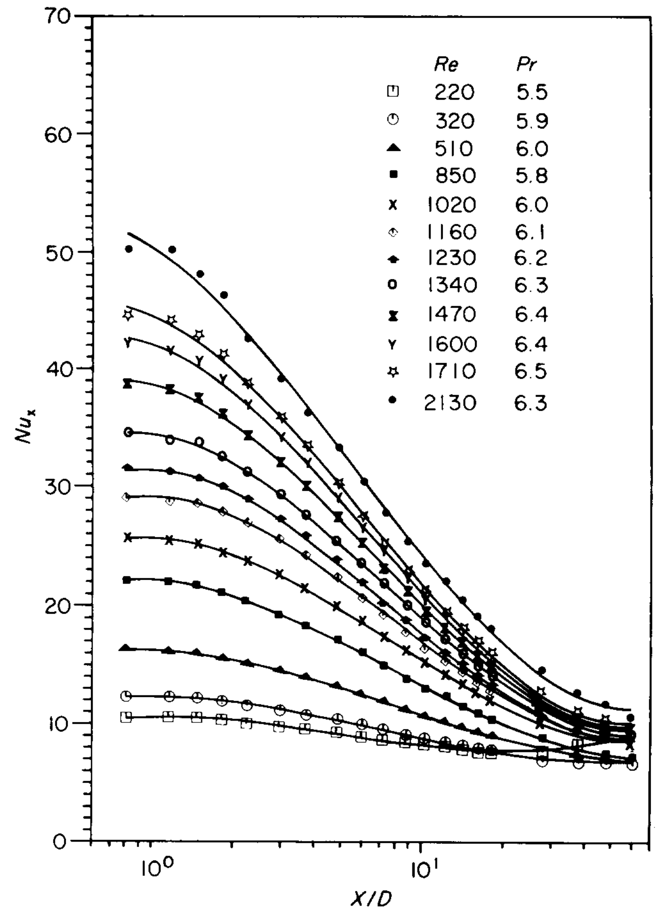

Fig 4 Local Nusselt number (Nu_x) *versus the dimensionless distance* (x/D) *from the entrance* (Pr ≅ *4*)

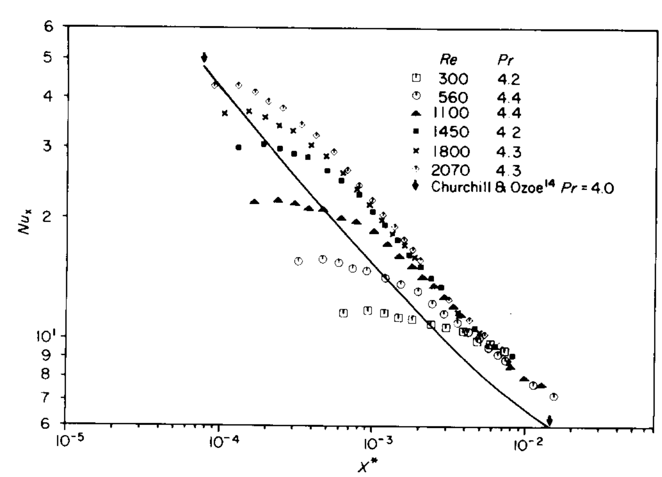

Fig 5 Local Nusselt number (Nu_x) *versus dimensionless distance* (x^*) *from the entrance* (Pr ≅ *4*)

8%, except in the first diameter length where the longitudinal flux could not be evaluated with sufficient accuracy. Consequently, the experimental error on the first point ($x/D = 0.8$) may be greater than 8%. The internal heat generation is negligible, as water is used and, according to the criterion suggested by Hennecke[19], no axial conduction along the fluid should be present since the Peclet number is always high.

- The effect of viscosity variation has been evaluated by both the corrective coefficient $(\mu_f/\mu_w)^{0.14}$ of the Sieder and Tate formula[20], and the Yang criterion[21]. An increase of Nu_x, generally lower than 2%, with maximum values of about 8% for some points near the end of the heated section, has been found.
- Attempts have been made to account for the influence of free convection on the heat transfer by means of the Eckert and Diaguila equation (see reference 22, Eq (10)) and the Metais and Eckert diagram[23], but unreliable results have been obtained. The following equation was given by Petukhov *et al*[24] to correlate their experimental data on the laminar mixed convection in pipes:

$$\frac{Nu_x}{Nu_c} = \left[1 + \frac{Ra}{B}\right]^{0.27} \tag{3}$$

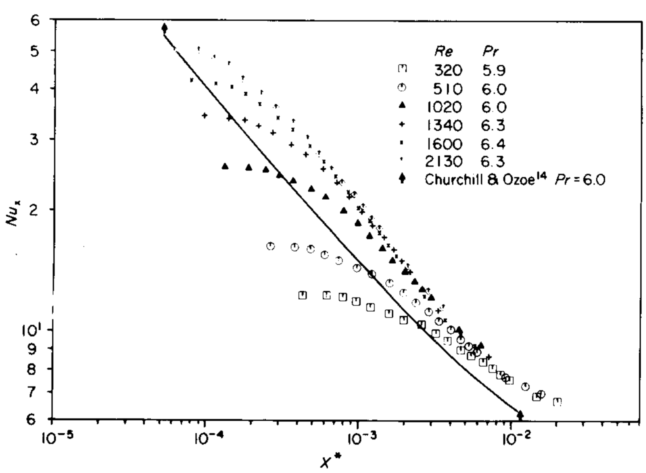

Fig 6 Local Nusselt number (Nu_x) *versus dimensionless distance* (x^*) *from the entrance* ($Pr \cong 6$)

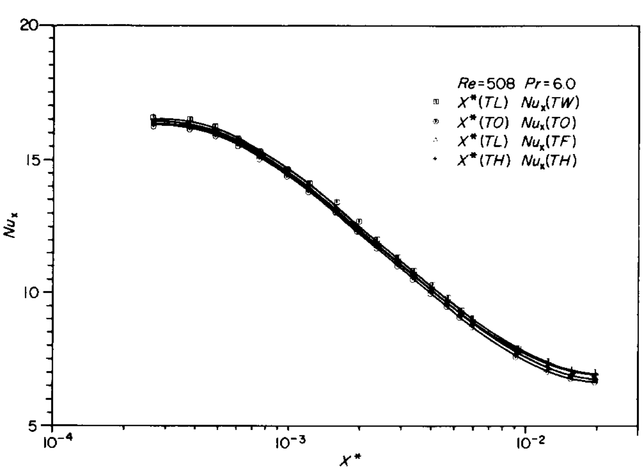

Fig 7 Influence of the choice of the reference temperature

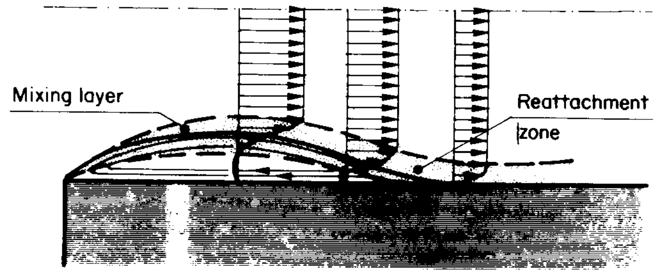

Fig 8 Flow configuration at the inlet of the pipe

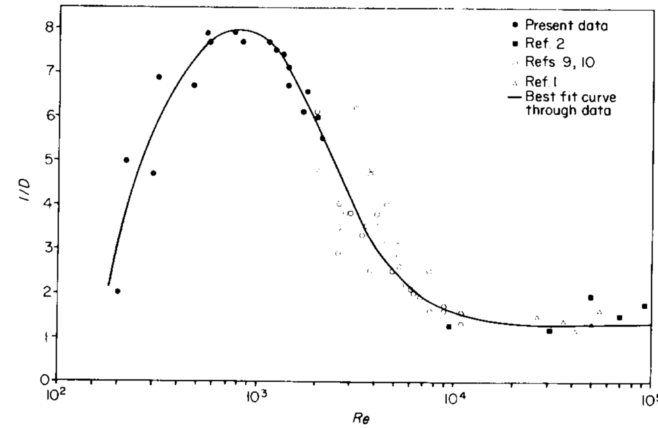

Fig 9 Position of point of inflexion versus Reynolds number

where:

$$B = 1.85(x^*)^{-1} + 78(x^*)^{0.25} \quad \text{if } x^* \leq 0.07$$

$$B = 60 \quad \text{if } x^* > 0.07$$

A maximum increase of 15% on the right side of Eq (3) has only been found for a few points at $Pr = 4$. Both natural convection effects and viscosity variation are negligible near the inlet section.

- For all the tests performed, Nu_x and x^* have been evaluated at various reference temperatures:

TH: Nu_x and x^* at the bulk temperature at half of the heated length $[Nu_x[TH], x^*(TH)]$;

TO: Nu_x and x^* at the inlet fluid temperature $[Nu_x(TO), x^*(TO)]$;

TL: x^* at the local bulk temperature and Nu_x at the local wall temperature $[Nu_x(TW), x^*(TL)]$;

TF: x^* at the local bulk temperature and Nu_x at the local film temperature $[Nu_x(TE), x^*(TF)]$. The local film temperature is the arithmetical mean value of the bulk and wall temperature.

It may be seen from Fig 7, where a single test is considered as an example, that the results are not strongly affected by the choice of the reference temperature. Criteria *TH* and *TL* give almost equivalent results while differences of 2–8% are found when *TO* and *TF* are employed. The present results are based on the *TH* reference temperature.

From this discussion it is clear that neither the choice of the reference temperature nor the variability of physical properties may be held completely responsible for the deviation between experimental and theoretical heat transfer coefficients. However, viscosity and density changes presumably affect the results far from the entrance, where the expected tendency towards the theoretical asymptotic value of Nu_x does not appear.

The trend of the data might be better understood when flow separation is assumed to occur at

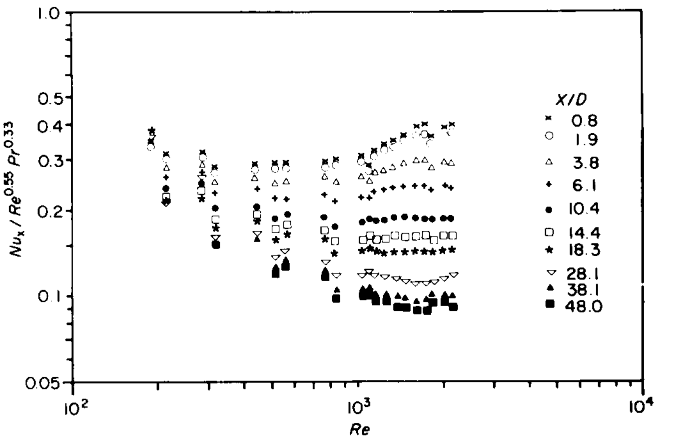

Fig 10 Nu/Re$^{0.55}$Pr$^{0.33}$ *versus Reynolds number at various values of* x/D (*laminar flow*)

the inlet section of the pipe. According to the Chapman model of laminar separation[25], the development of a shear layer starting at a sharp edge (Fig 8) causes very low values of the Nusselt number, when the heat transfer from an isothermal surface to the separated layer is considered. It is noteworthy that Nu_x is seen to be an increasing function of the Prandtl number in this case.

As the core of the mixing layer, characterised by a large velocity gradient, is brought into contact with the wall, very high heat transfer rates should be found[26], whereas thereafter a regular development of the boundary layer should take place, so that from this point the heat transfer is controlled by a new mechanism. The trend of the present results (see figs 3 and 4) conforms to this model, showing an initial region corresponding to the separated flow region, with negative concavity, followed by the boundary layer redevelopment zone, characterised by the usual positive concavity.

It may be observed that the results of Ota and Kon[27] and Smyth[28], dealing with the effect of separation on the heat transfer in external turbulent flow, qualitatively agree with those of Mills[2] and Ede *et al*[6]. They show an initial region where Nu_x is very

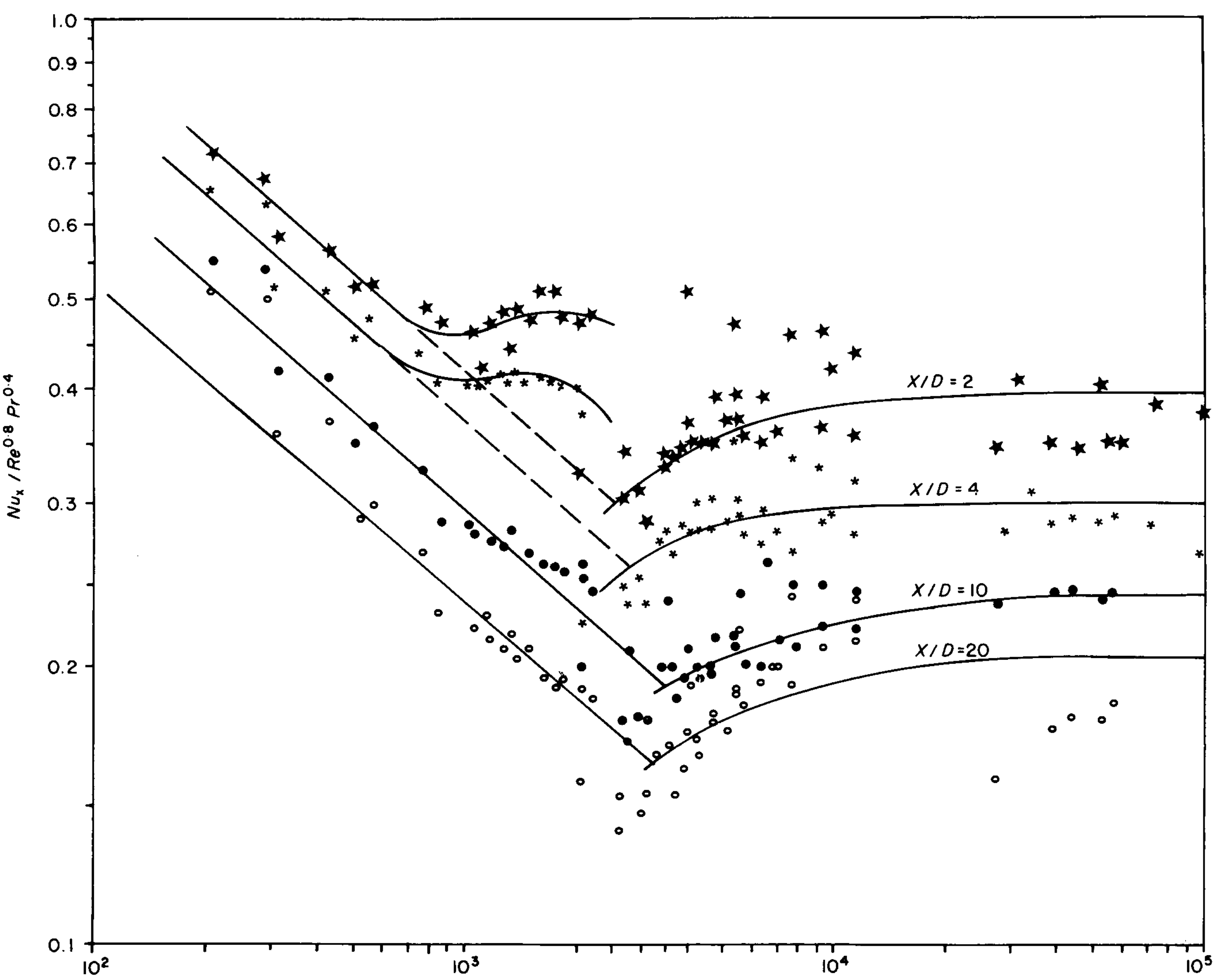

Fig 11 Nu/Re$^{0.8}$Pr$^{0.4}$ *versus Reynolds number at various values of* x/D (*general*)

low followed by an increase with the downstream distance from the separation point. After reaching a maximum, Nu_x decreases to an asymptotic value. The length of the separation region is found to be independent of the Reynolds number[27,28]. Thus it seems possible to obtain an indication of the separation length from the coordinate of the point of inflexion, on the Nu_x versus x/D plots.

To investigate the dependence of the separation length on Re, the logarithmic polynomial best-fit was found for both the data presented here and other workers' results[1,2,9,10]. The dimensionless distance, l/D, of the point of inflexion from the inlet was then computed and is plotted against the Reynolds number in Fig 9. The increase of the value of l/D for $Re < 1000$ agrees with the observations of Hurd and Peters[29], who considered the laminar flow inside a constant width channel with a sharp 90° turn for Re as high as 800. They found that the separated region downstream of the turn lengthens with increasing Reynolds number. After the maximum, at about $Re = 1000$, the length of the separated zone decreases, probably due to the onset of fluid-dynamic instabilities which tend to reduce its length, towards an asymptotic value of about 1.5 for highly turbulent flow.

General correlations

The general effect of an abrupt convergence may be summarized by a cross-plot of Nu/Re^aPr^b against Re for various values of x/D. The results for laminar flow, or rather for $Re < 2000$, are plotted in Fig 10, according to the correlating equation given elsewhere[13]; the values of a and b are 0.55 and 0.33 respectively.

From the previous results[1,2,9,10] and the present data, a general plot (Fig 11) has been obtained. The values $a = 0.8$ and $b = 0.4$ were used, following the suggestion of Ede *et al*[6]. The solid lines represent a least-squares fit of all these results and provide a practical tool to predict the local heat transfer coefficient in a pipe, following an abrupt convergence, over the entire range of Re.

For Re as high as about 800 and for Re greater than 20 000 a rather regular dependence of Nu_x on the Reynolds number is found for all values of x/D. A large scatter of data is shown for $2000 < Re < 16\,000$; nevertheless the typical trend of the transitional flow results may be inferred from them. For $800 < Re < 2000$, at the low values of x/D, the results vary little with Re. This suggests that the first onset of fluid-dynamic instabilities might take place immediately after the inlet, at about $Re = 1000$.

Conclusions

Present measurements illustrate the effect of an abrupt convergence with a high contraction ratio on the local heat transfer coefficient in laminar flow at the entrance of a straight circular pipe. The results show that the ordinary theory of the development of the boundary layer is not valid for this geometrical configuration, while they agree with the hypothesis of the presence of a separated flow at the inlet section of the pipe.

These and several previous experiments extending into turbulent flow, have been analysed on the basis of this hypothesis, and the separation length has been evaluated. It has been found that this length is strongly Reynolds number dependent in laminar and transitional flow and assumes a maximum value in the region of $Re = 1000$.

Acknowledgements

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BOOK REVIEWS

Heat Transfer

J. P. Holman

The first edition of this elementary treatment of the principles of heat transfer was published in 1963; that the author has made strenuous efforts to keep it up to date is demonstrated by the appearance of successive editions at intervals not exceeding five years. One of the stated objectives of this fifth edition is to include recent information about analytical techniques and experimental data while still retaining a simple approach which can be understood by the student at junior grade level or above. It is, however, difficult to accept the implication that with this additional information the material can still be adequately covered in a one-semester course.

The sections on numerical solutions to multidimensional and unsteady-state conduction problems have been expanded. New and useful examples have been added illustrating treatments of radiation and non-linear convection boundary conditions, variable properties, non-uniform nodal spacings, composite materials and iterative-solution techniques for sets of non-linear equations.

The chapters on forced and free convection have been updated in regard to empirical correlations, though the treatment of high-speed flow is more restricted than might have been expected; moreover, the attention given to turbulence, usually in terms of fluid instability, the time-averaged equations and Reynolds stresses, seems quite inadequate for a proper understanding of the complex processes of convection unless the student has some familiarity with fluid mechanics. Thus I find myself in disagreement with the author's contention that such familiarity is not essential to a fruitful study of his book.

Some new examples using both the network and matrix solution techniques are introduced in the chapter on radiation heat transfer. These include iterative solutions for non-linear problems involving both convection and radiation, as in the topical case of a solar heat collector. It is disappointing to find that the additions to the chapter on condensation and boiling pay so little attention to the effects of non-condensable gases on condensation, and particularly to the long-standing measurements of Hampson and Ozesek[1], and that Kutataladze's[2] criterion for the peak heat flux in nucleate pool boiling is ignored, the more so since it also readily covers subcooled boiling.

Additional analytical expressions for heat-exchanger performance parameters are coupled with a new section on compact heat exchangers. This might well have included those incorporating heat pipes, one of the special topics discussed in a later chapter, and though updated, still without reference to the dimensionless wick properties and liquid transport factors which help to determine performance, or to the capillary and boiling restrictions and the sonic limitation on mass flow. Particularly welcome are the new examples to illustrate energy storage systems and variable-property analysis in a simple heat exchanger.

Emphasis in the numerical-solution sections has been placed on problem formulation rather than on specific computer programs to solve heat transfer problems. In so choosing, the author is to be commended in anticipating the use of desk-top minicom-